Tutorial 9 09, Nov. 2015.

1. If f is a holomorphic function on the strip $-1 < y < 1$, $x \in \mathbb{R}$ with
$$|f(z)| \le A(1+|z|)^{\eta}, \qquad \eta \text{ a fixed real number,}$$
for all z in that strip. Show that for each integer $n \ge 0$ there exists $A_n \ge 0$ so that
$$|f^{(n)}(x)| \le A_n (1+|x|)^{\eta} \text{ for all } x \in \mathbb{R}.$$

2. Suppose $f: \mathbb{D} \to \mathbb{C}$ is holomorphic. Show that the diameter d
$d = \sup_{z,w \in \mathbb{D}} |f(z) - f(w)|$ of the image of f satisfies
$$2|f'(0)| \le d.$$
~~Hint~~ [Hint: $2f'(0) = \frac{1}{2\pi i} \int_{|\zeta|=r} \frac{f(\zeta) - f(-\zeta)}{\zeta^2} d\zeta$]

~~3.~~

3. Evaluate the integral
$$\int_{-\infty}^{+\infty} \frac{dx}{1+x^4}.$$